

# Modeling Business Cycles with Markov Switching Arma (Ms-Arma) Model: An Application on Iranian Business Cycles

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## Abstract

In this paper, the Iran Business Cycle characteristics were investigated via numerous univariate and multivariate Markov-switching specifications. In this case Markov switching model MSM-ARMA is proposed for determining business cycles. We examined the stochastic properties of the cyclical pattern of the quarterly Iran real GDP between 1988 (1) – 2008 (2). The empirical analysis consists of mainly three parts. First, a large number of alternative specifications were tried and few were adopted with respect to various diagnostic statistics. Then, all selected models were tested against their linear benchmarks. LR test results imply strong evidence in favor of the nonlinear regime switching behavior. In line with the main objective of research, proposed model for Iran business cycle is estimated by and result of this estimation showed that economic of Iran despite of having two periods of recession 1992(3) - 1992(4) and 1995(1)-1995(2), is out of recession with moderate growth and also experienced growth with high rate in early period of studying. Also the possibility of resistance of recession regimes with moderate and high growth is 0.3, 0.92 and 0.5 respectively. The results show the economic tend to stay in moderate growth regime.

**Keywords:** Markov Switching Models, Business Cycles, MS-ARMA, Iran Economy

**JEL Classifications:** E32, C32.

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## Introduction

Research on business cycles has always been at the core of economic research agenda where one of the pioneering studies on the topic belongs to Burns and Mitchell (1946). This tradition has opened up two research areas namely, co-movement among variables through the cycle, and the different behavior of the economy during different phases of the cycle. The first one gave rise to the formation of dynamic factor models and composition of indices. The latter one inspired the use of nonlinear regime switching models with the seminal work of Hamilton (1989) that addressed whether the asymmetric movements occur systematically enough to be counted as part of the probabilistic structure of time series. The underlying idea was that business cycle expansions and contractions could be viewed as different regimes. Two extensions of Hamilton (1989) model were Filardo (1994) and Diebold et al (1994). These models assume that the probability of regime switching may be dependent on underlying economic fundamentals. Recent research has witnessed a synthesis of co-movement and nonlinearity features of cycles since there is room for the analysis by incorporating both factor structure and regime switching (see Diebold and Rudebusch (1996) Chauvet (1998, 2001) and Kim and Nelson (1998) among others).

The harmonization of two different methods of business cycle analysis also gave rise to Markov-switching vector auto-regression (MS-VAR) models developed by Krolzig (1997). In these extended models there is an unobserved state driven by an ergodic Markov process that is common to all series. In subsequent studies, Clements and Krolzig (2002, 2003) discussed the characterization and the testing of business cycle asymmetries based on MS-VAR models. Pelagatti (2002) estimated a duration dependent MS-VAR model by using a multi-move Gibbs sampler since the computational burden in using the ML approach to such models is high. Ehrmann et al. (2003) combined both Markov-switching and structural identifying restrictions in a VAR model to analyze the reaction of variables to fundamental disturbances.

Despite these very influential recent developments both in theoretical and empirical literature, the analysis of Iran business cycles has been somewhat limited and concentrated heavily on the leading indicators approach.

Our major aim in this paper is to contribute in empirical modeling of Iran business cycles with the help of MS models. Of our particular concern is MS-ARMA models where the unobserved state is assumed to be common to all series used in model specifications. We consider both the co-movement and the nonlinearity of the cyclical process of Iran economy by employing a variety of MS-ARMA models in which some or all of the parameters are allowed to change with the regime. Even though, our concern is on the determination of business cycle turning points. The paper is organized as follows. Section 2 describes the various specifications of MS-ARMA model and the estimation process via EM algorithm. Section 3 gives a brief overview of the pertinent events of Iran economy in the considered period. Section 4 introduces the data set and presents the empirical results obtained from the application of various MS-ARMA models to univariate and multivariate time series. The final section concludes.

## Markov Switching Auto Regressive Integrated Moving Average (Ms - Arma)

Markov-Switching ARMA (MS-ARMA) processes are a modification of the well-known ARMA processes by allowing for time-dependent ARMA coefficients, which are modeled as a Markov chain. We will first review the MS-ARMA class of models and then continue with the estimation process via the EM algorithm

### The Model

MS-ARMA class of models provides a convenient framework to analyze representations with changes in regime. They admit various dynamic structures, depending on the value of the state variable,  $s_t$ , which controls the switching mechanism between various states. In these models, some or all of the parameters may become varying with regard to the regime prevailing at time  $t$ . Besides, business cycles are treated as common regime shifts in the stochastic processes of macroeconomic time series. In other words, both nonlinear and common factor structures of the cyclical processes are represented at the same time. Consider the MS-ARMA process in its most general form(1).

$$\Delta y_t = V(S_t) + \sum_{i=1}^p A_i \Delta y_{t-i} + \sum_{j=1}^q B_j \varepsilon_{t-j} + \varepsilon_t \quad (1)$$

Where,  $S_t$  stands for the state at time  $t$  - there is a finite number of states;  $P$  is the number of autoregressive elements;  $V$  is the vector of intercepts, which varies with the states;  $A_i$  shows the autoregressive coefficients for lags 1 to  $p$  for each state,  $B_j$  shows the Moving Average coefficients for lags 1 to  $q$  for each state and  $\varepsilon_t$  are the residuals characterized by a zero mean and a variance equal to 1.  $\varepsilon_t | s_t \sim NID(0, \Sigma(s_t))$ . The MS-ARMA setting also allows for a variety of specifications. Table 1 gives an overview of the MS-ARMA models.

Table 1: Types of MS-ARMA Models

Notation	$\mu$	$v$	$\Sigma$	$A_i$
MSM(M)-ARMA(p)	varying	-	invariant	invariant
MSMH(M)-ARMA(p)	varying	-	varying	invariant
MSI(M)-ARMA(p)	-	varying	invariant	invariant
MSIH(M)-ARMA(p)	-	varying	varying	invariant
MSIAH(M)-ARMA(p)	-	varying	varying	varying

$\mu$  : mean,  $v$  : intercept  $\Sigma$  : variance  $A_i$  : matrix of autoregressive parameters

In Equation 1 the intercept term is assumed to vary with state beside other parameters. Intercept switch specification is used in cases where the transition to the mean of the other state is assumed to follow a smooth path. An alternative representation is obtained by allowing the mean to vary with the state. This specification is useful in cases where a one-time jump is assumed in the mean after a change in regime.

The description of the dynamics is complete after defining a probability rule of how the behavior of  $y_t$  changes from one regime to another. Markov chain is the simplest time series model for a discrete-valued random variable such as the unobserved state variable  $s_t$ . In all MS-ARMA specifications it is assumed that the unobserved state  $s_t$  follows a first-order Markov-process. The implication is that the current regime  $s_t$  depends only on the regime one period ago,  $s_{t-1}$  (2)

$$P\{s_t = j | s_{t-1} = i, s_{t-2} = k, \dots\} = P\{s_t = j | s_{t-1} = i\} = p_{ij} \quad (2)$$

where  $p_{ij}$  gives the probability that state  $i$  will be followed by state  $j$ .

These transition probabilities can be collected in a  $(N \times N)$  transition matrix, denoted as  $P$ . Each element in the transition matrix  $p_{ij}$  represents the probability that event  $i$  will be followed by event  $j$ .

$$P = \begin{bmatrix} p_{11} & p_{21} & \dots & p_{N1} \\ p_{12} & p_{22} & \dots & p_{N2} \\ \vdots & \vdots & \dots & \vdots \\ p_{1N} & p_{2N} & \dots & p_{NN} \end{bmatrix}$$

with

$$\sum_{j=1}^N p_{ij} = 1 \quad \text{where } i = 1, 2, \dots, N \quad \text{and} \quad 0 \leq p_{ij} \leq 1$$

For a two-state case, we can represent the transition probabilities by a  $(2 \times 1)$  vector,  $\hat{\xi}_{t|t}$ , whose first element is  $P(s_t = 1 | \psi_t)$  where  $\psi_t = \{\psi_{t-1}, y_t\}$  and  $\psi_{t-1}$  contains past values of  $y_t$ . If we know the value  $\hat{\xi}_{t-1|t-1}$ , then it would be straightforward to form a forecast of the regime for  $t$  given the information at  $t-1$  and collect the terms for the probabilities of  $s_t = 1, 2$  in a vector denoted by  $\hat{\xi}_{t|t-1}$  as follows:

$$\hat{\xi}_{t|t-1} = \begin{bmatrix} P(s_t = 1 | \psi_{t-1}) \\ P(s_t = 2 | \psi_{t-1}) \end{bmatrix}$$

We can specify the probability law of the observed variable  $y_t$  conditional on  $s_t$  and  $\psi_{t-1}$  and collect them in a  $(2 \times 1)$  vector  $\eta_t$ :

$$\eta_t = \begin{bmatrix} f(y_t | s_t = 1, \psi_{t-1}) \\ f(y_t | s_t = 2, \psi_{t-1}) \end{bmatrix}$$

The joint probability of  $y_t$  and  $s_t$  is then given by the product

$$f(y_t, s_t = j | \psi_{t-1}) = f(y_t | s_t = j, \psi_{t-1}) P(s_t = j | \psi_{t-1}), \quad j = 1, 2$$

The conditional density of the  $t$ th observation is the sum of these terms over all values of  $s_t$ . For a two-state case:

$$f(y_t | \psi_{t-1}) = \sum_{s_t=1}^2 \sum_{s_{t-1}=1}^2 f(y_t | s_t, \psi_{t-1}) P(s_t | \psi_{t-1}) = \eta_t' \hat{\xi}_{t/t-1}$$

Then, the

output  $\hat{\xi}_{t/t}$  can be obtained from the input  $\hat{\xi}_{t-1|t-1}$  by following the steps described in Hamilton (1994, Chapter 22).

### *Estimation*

The conventional procedure for estimating the model parameters is to maximize the log-likelihood function and then use these parameters to obtain the filtered and smoothed inferences for the unobserved state variable  $s_t$ . However this method becomes disadvantageous as the number of parameters to be estimated increases. Generally in such cases, the Expectation Maximization (EM) algorithm, originally described by (Dempster et al. 1977) is used. This technique starts with the initial estimates of the hidden data and iteratively produces a new joint distribution that increases the probability of observed data. These two steps are referred to as expectation and maximization steps. The EM algorithm has many desirable properties as stated in (Hamilton, 1990).

### **A Brief Account of the Iran Economy and Business Cycles**

One of the indexes reflecting the amount of economic activity in any country is Gross Domestic Product which is the total value of final goods and services produced domestically during a period (in a single year). As it can be seen, up to 1974 the GDP has been picking up and after that when Islamic revolution happened its rate of growth has decreased to -7.3. By 1979, the negative rate of growth continues due to intensified political problems and issues such as the import and export markets being closed. In 1980 and 1981, the positive rate of growth up to 12.5 and 11.06 is clear respectively due to reopening of the

factories and the government being put in its right position. From 1984 till the end of 1988, because import and export market was closed and also due to the sanctions on Iran by other countries, decrease in the oil price in the world market, GDP declined severely by negative rates of growth during 1986, 1987, and 1988. Later on, by the government's change of policies for export and import and being incorporated in the first development plan (1989-1993), the production market flourishes again and GDP experiences a high level of growth. The increase in the growth rate of GDP to 12.12 in 1991 was because of the Iraq-Kuwait crisis and a consequent increase in oil price in the world market. Following that, from 1993 to 1995 the rate of growth decreased again in a way that in 1994 it became 0.49 percent. After that period, the rate of growth reached 8.1 in 2002. This rate declined to 3.4 due to the sanctions imposed on Iran and an increase in the imports from China and a decrease in domestic production. As is evident, the instability of the GDP growth has been the main indicator of the cyclical pattern of the Iran economy. This points out to the need for rigorous empirical modeling of the Iran business cycles. Next section presents the results obtained from the application of a variety of MS-ARMA specifications to capture the cyclical dynamics of the Iran Economy during the period under consideration

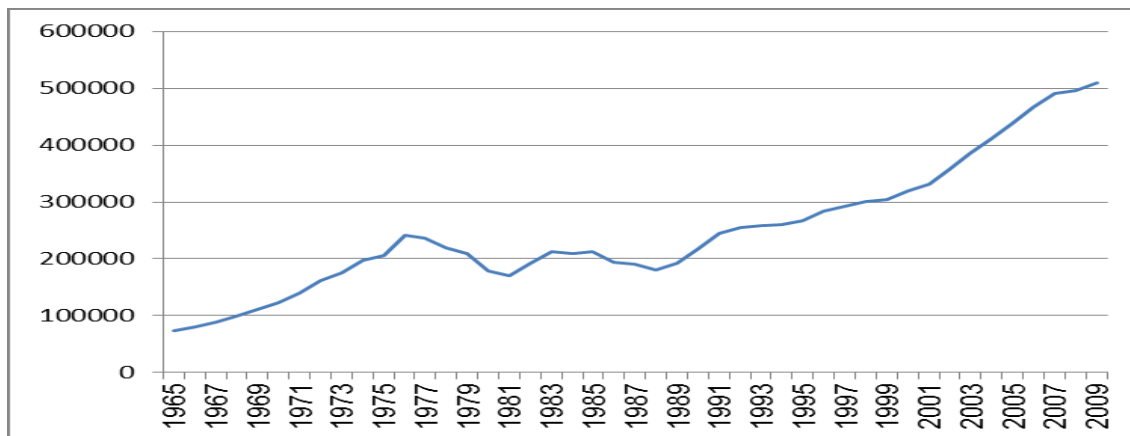


Figure 1. GDP at Fixed Price in Billion Rials (National Currency) from 1965 to 2009

## Empirical Results

In this section we will present the results of the econometric specifications used for modeling the Iran business cycles between 1988 and 2008. We will begin by introducing the data set and the results from the model selection procedure. Then, we will interpret the findings and compare the predictive performances of the alternative models.

### *Data Analysis*

In the empirical analysis, The study data indicates GDP based on 1997 as a base year which has been extracted seasonally from the Central Bank of the Islamic Republic of Iran

RGDP is seasonally adjusted using a multiplicative moving average method. In order to achieve stationary, one hundred times natural logarithms of the first differences of the series are used (Figure 2).

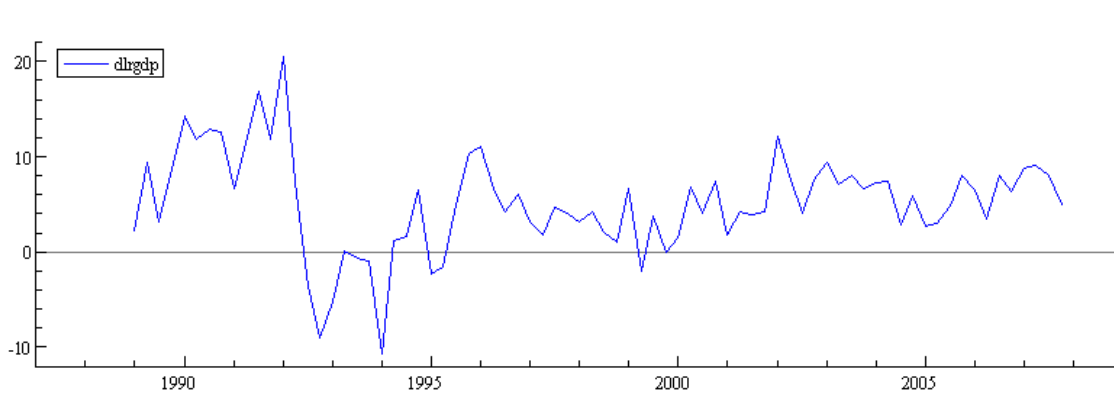


Figure 2 The variable under analysis\*

\* Percentage changes in the variables calculated as hundred times log difference.

### *Choosing the appropriate MS specifications for the Iran Business Cycles*

Our model selection process consists of two steps. In the first step, for choosing among different MS specifications, Akaike Information (AI), Hannah-Quinn (HQ) and Schwarz (SC) criteria are used. The alternative specifications were MS models with mean, intercept and variance coefficients that are allowed to switch across regimes. Then, all models are tested for linearity by taking the linear model as the null hypothesis and the regime-switching model as the alternative. We applied these selection criteria both for univariate and MS ARMA Models. Laurent (2003) consider a three-regime MS model with the following economic interpretation.

- low regime ( $St = 1$ ): the economy is in recession (low phase of the business cycle)
- intermediate regime ( $St = 2$ ): the growth rate of the economy is below its trend growth rate (low phase of the growth cycle without recession)
- high regime ( $St = 3$ ): the growth rate of the economy is over its trend growth rate (high phase of the growth cycle)

The transition between states is characterized by a first order Markov chain and duration independency is also assumed. For model selection, a mean switch model, an intercept switch model with changing variance and a MS-ARMA(p,q) model are estimated using RGDP for the period from 1988:Q2 through 2008:Q2. Since Markov-switching models are produced by switching autoregressive model in mean, intercept, and autoregressive coefficients, for selecting optimal model, Akaike value should be minimum and the null hypothesis (H0) of no regime switching in the model can be rejected. Given the test statistic values as well as of error normality hypotheses, comparing three-regime models are yielded better results than the two-regime models. The results of H0 testing and Akaike criterion at three-regime state are summarized in table 2. Results indicated that MSIMH (3)-ARMA (5,2) model is selected as an optimal model which the normality hypothesis is confirmed and has a larger maximum likelihood.

Table2: The results of H0 testing and Akaike criterion at three-regime state

	MSI(2) GDP(4)	MSM(2) ARMA(2,3)	MSMA(2) ARMA(3,4)	MSMH(3) ARMA(2,1)	MSMH(3) ARMA(4,2)	MSMH(3) ARMA(5,1)	MSMH(3) ARMA(5,2)
Akaike Information(AIC)	5.679	5.684	5.584	5.738	5.713	5.451	5.665
Maximum likelihood	-200.137	-205.021	-189.413	-202.061	-192.387	-179.96	-186.782
Normality test	Chi <sup>2</sup> (2) 5.08 [0.07]	Chi <sup>2</sup> (2) 5.521 [0.06]	Chi <sup>2</sup> (2) 5.19 [0.07]	Chi <sup>2</sup> (2) 1.604 [0.44]	Chi <sup>2</sup> (2) 0.424 [0.80]	Chi <sup>2</sup> (2) 2.515 [0.28]	Chi <sup>2</sup> (2) 1.55 [0.45]
Heteroscedastic Test	F(1,62) 0.753 [0.38]	F(3,59) 0.142 [0.93]	F(4,47) 0.518 [0.72]	F(3,54) 0.887 [0.45]	F(5,45) 0.551 [0.73]	F(6,42) 0.438 [0.84]	F(6,41) 0.329 [0.91]
Autocorrelation Test	Chi <sup>2</sup> (8) 6.876 [0.55]	Chi <sup>2</sup> (12) 13.04 [0.36]	Chi <sup>2</sup> (12) 11.32 [0.50]	Chi <sup>2</sup> (12) 6.75 [0.87]	Chi <sup>2</sup> (12) 7.675 [0.80]	Chi <sup>2</sup> (12) 11.44 [0.49]	Chi <sup>2</sup> (12) 7.017 [0.85]

In order to test between linearity versus non-linear regime switching specifications a testing procedure developed by Ang and Bekaert (2001) is used. In this paper it is suggested that the underlying distribution can be approximated by a  $\chi^2(q)$  distribution where  $q$  represents the number of restrictions and nuisance parameters that are not defined under the null hypothesis.

All of the above presented estimation statistics and the results of linearity tests highlight the need for nonlinear models to characterize cyclical dynamics. In the light of this finding, we will proceed with the estimation results of the MS models and their implications for the cyclical structure of Iran economy.



Table3: LR test for nonlinearity test data, the GDP growth

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LR-test $\chi^2(3) = 15.007$ [0.0018]
LR-test $\chi^2(8) = 34.569$ [0.0000]

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### Comments on Estimated MS Models

Table 4 reports the maximum likelihood estimates of MS models obtained by the EM algorithm. For the MSMH (3)-ARMA (5,2) model, refers to the average growth rate of quarterly RGDP series in state 1 with the economy is in recession(-3.92) is the average growth rate of RGDP in state 2 with intermediate regime(4.43) is the average growth rate of RGDP in state 3 with the growth rate of the economy is over its trend growth rate (9.57). For all other models the intercept, instead of the mean is assumed to be state dependent. mean switch model, the estimated quarterly growth rate is 4.43 % in expansions and -3.92 % in recessions. This result points out to the volatility of output growth during periods of recessions and expansions.

For Iran ,more robust statistical results are obtained if the model is extended to allow the series to switch among three different economic regimes, as well as assuming regime-dependent intercepts, autoregressive components and heteroskedastic errors (i.e. the MSMH(3)-ARMA(5,2) specification). The three regimes can be attributed to different economic phases, namely null, moderate and high economic growth, with the first regime characterizing the periods 1992(3) - 1992(4), 1994(1)-1994(1), 1995(1) - 1995(2), 1999(4) - 1999(4) and 2000(3) - 2000(3).(Table 6)

. According to this model, regime 1 tends to last 7 quarters on average, while regime 2 is 56 quarters. Finally, high growth periods tend to last 10 quarters on average. Transition probabilities of regimes are 0.3 for regime 1, 0.92 for regime 2 and 0.49 for regime 3.(table 5 ) Optimal inferences of turning points are obtained from the smoothed probabilities of the Markov states. Due to the decision rule proposed by Hamilton (1989), if  $P(s_t = 1 | \psi_T) > 0.5$ , the economy is in a recession, otherwise it is in an expansion. Figure 3, 4 gives a graphical display of the filtered and smoothed probabilities of regime 1 produced by all four models. Smoothed probabilities of all models display that downswings are abrupt and much shorter while upswings are more gradual and highly persistent.

Table 4: The results of estimating the model parameters to optimize the business cycles

Variable	Coefficients	Standard deviation	Statistics t
$\mu_0$	-3.92	2.48	-1.65
$\mu_1$	4.43	2.42	1.83
$\mu_2$	9.57	2.18	4.37
AR-1	0.77	0.06	12.8
AR-2	-0.11	0.05	-2.1
AR-3	0.61	0.04	13.3
AR-4	-0.48	0.06	-7.06
AR-5	0.07	0.079	0.988
MA-1	-0.006	0.05	-0.115
MA-1	0.0047	0.022	0.161
dumm1371	6.70	0.47	18.1
Sigma(0)	4.14	1.22	4.48
Sigma(1)	2.63	0.25	10.4
Sigma(2)	0.4	0.1	4.04
log-likelihood	-186.78		
AIC	5.66		
SC	6.29		
HA	5.91		
Normality Test	Chi <sup>2</sup> (2)=1.55[0.45]		
ARCH Test	F(6,41)=0.42[0.91]		
Portmanteau statistic for Autocorrelation residuals	Chi <sup>2</sup> (12)=7.01[0.85]		

Table5: Transition probabilities and Duration Regime for Business cycles of Iran

	Low Regime and period t	Intermediate Regime and period t	High Regime and period t
Low Regime and period t+1	0.29576	0.00	0.5042
Intermediate Regime and period t+1	0.70424	0.91745	0.00
High Regime and period t+1	0.00	0.08265	0.4958
Duration(Quarter)	7	56	10

Table 6: Dating Iran Business Cycle Turning Points Using Smoothed Probabilities

Low Regime	Intermediate Regime	High Regime
1992(3) - 1992(4)	1990(2) - 1991(2)	1991(3) - 1992(2)
1994(1)-1994(1)	1993(1) - 1993(1)	1993(2) - 1993(4)
1995(1) - 1995(2)	1994(2) - 1994(3)	1994(4) - 1994(4)
1999(4) - 1999(4)	1995(3) - 1999(2)	1999(3) - 1999(3)
2000(3) - 2000(3)	2000(1) - 2000(1)	2000(2) - 2000(2)
	2000(4) - 2008(2)	

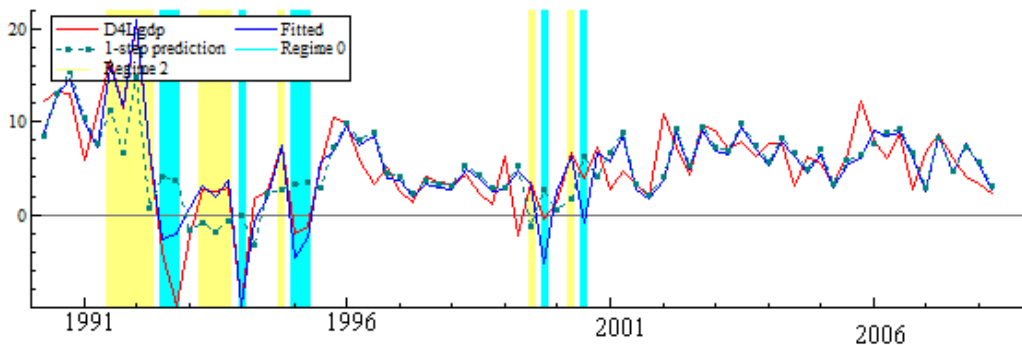


Figure 3: Fit of the MSMH(3)-ARMA(5,2) Model for RGDP

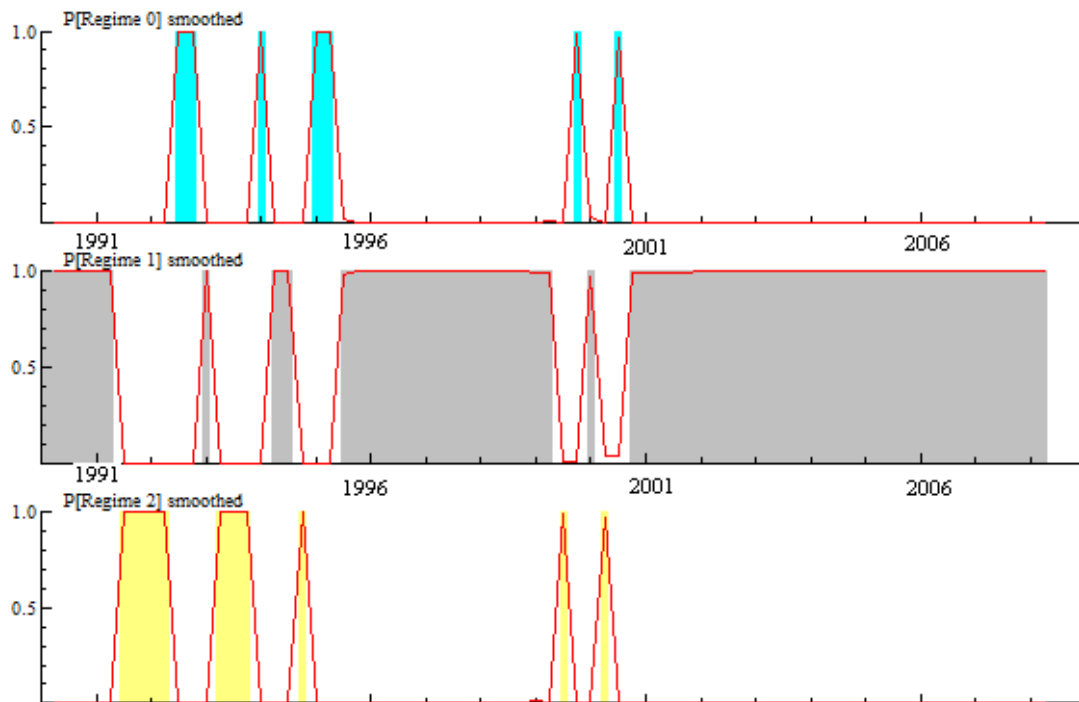


Figure 4: Filtered and Smoothed Probabilities of Regime 1, 2, 3 for Model

## CONCLUSION

In this paper, we employed various specifications of MS-ARMA models to empirically characterize the state dependent dynamics of the Iran business cycles between 1988 and 2008. Our findings can be summarized as follows. Linearity of GDP series is severely rejected implying that there is regime switching structure in Iran business cycles. In line with the main objective of research, proposed model for Iran business cycle is estimated by and result of this estimation showed that economic of Iran despite of having two periods of recession 1992(3) - 1992(4) and 1995(1)-1995(2), is out of recession with moderate growth and also experienced growth with high rate in early period of studying. Also the possibility of resistance of recession regimes with moderate and high growth is 0.3, 0.92 and 0.5 respectively. The results show the economic tend to stay in moderate growth regime.

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